Op Amp Precision Design: Random Noise

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INTRODUCTION

This application note covers the essential background information and design theory needed to design low noise, precision op amp circuits. The focus is on simple, results oriented methods and approximations useful for circuits with a low-pass response.

The material will be of interest to engineers who design op amps circuits which need better signal-to-noise ratio (SNR), and who want to evaluate the design trade-offs quickly and effectively.

This application note is general enough to cover both voltage feedback (VFB) (traditional) and current feedback (CFB) op amps. The examples, however, will be limited to Microchip's voltage feedback op amps.

Additional material at the end of this application note includes references to the literature, vocabulary and computer design aids.

Key Words and Phrases

• Op Amp
• Device Noise
• Noise Spectral Density
• Integrated Noise
• Signal-to-Noise Ratio (SNR)

Prerequisites

The material in this application note will be much easier to follow after reviewing the following statistical concepts:

• Average
• Standard Deviation
• Variance
• Gaussian (normal) probability density function
• Histograms
• Statistical Independence
• Correlation

Knowledge of basic circuit analysis is also assumed.

BACKGROUND INFORMATION

This section covers the basics of low frequency noise work. It is somewhat theoretical in nature, but has some numerical examples to illustrate the concepts. It serves as a foundation for the following sections. See references [2, 4, 5] for a more in depth theoretical coverage of these concepts.

The material after this section illustrates these concepts. For those readers new to this subject matter, it may be beneficial to read the complete application note several times, while working all of the examples.

Where Did the Average Go?

The most commonly used statistical concept is the average. Standard circuit analysis gives a deterministic value (DC plus AC) at any point in time. Once these deterministic values are subtracted out, the noise variables left have an average of zero.

Noise is interpreted as random fluctuations (a stochastic value) about the average response. We will deal with linear circuits, so superposition applies; we can add the average and the random fluctuations to obtain the correct final result.

Noise Spectral Density

The easiest approach to analyzing random analog noise starts in the frequency domain (even for engineers that strongly prefer the time domain). Stationary noise sources (their statistics do not change with time) can be represented with a Power Spectral Density (PSD) function.

Because we are analyzing analog electronic circuits, the units of power we will deal with are W, V^2/Ω and $A^2\Omega$. This noise power is equivalent to statistical variance (σ^2). The variance of the sum of uncorrelated random variables is:

EQUATION 1: VARIANCE OF THE SUM OF UNCORRELATED VARIABLES

$$var\left(\sum_k X_k\right) = \sum_k var(X_k)$$

Where:

| X_k | = | uncorrelated random variables |
| $var()$ | = | the variance function |

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This fact is very important because the various random noise sources in a circuit are caused by physically independent phenomena. Circuit noise models that are based on these physically independent sources produce uncorrelated statistical quantities.

The PSD is an extension of the concept of variance. It spreads the variation of any noise power variable across many frequency bins. The noise in each bin (power with units of Watts) is statistically independent of all other bins. The units for PSD are (W/Hz), which is why it is called a "density" function. The picture in Figure 1 illustrates these concepts.
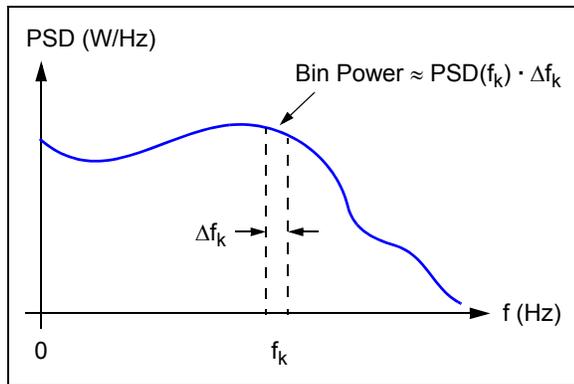

FIGURE 1: *Power Spectral Density.*

In this application note, all PSD plots (and functions) are one-sided, with the x-axis in units of Hertz. This is the traditional choice for circuit analysis because this is the output of (physical) spectrum analyzers.

Note:	It is very important, when reading the electronic literature on noise, to determine:
	• Is the PSD one-sided or two-sided?
	• Is frequency in units of Hertz (Hz) or Radians per Second (rad/s)?

In most low frequency circuits, signals and noise are interpreted and measured as voltages and currents, not power. For this reason, PSD is usually presented in two equivalent forms:

• Noise voltage density (e_n) with units (V/√Hz)
• Noise current density (i_n) with units (A/√Hz)

The voltage and current units are RMS values; they could be given as (V_{RMS}/√Hz) and (A_{RMS}/√Hz). Traditionally, the RMS subscript is understood, but not shown.

Note:	Many beginners find the √Hz units to be confusing. It is the natural result, however, of converting PSD (in units of W/Hz) into noise voltage or current density via the square root operation.

Strictly speaking, in passive circuits (RLC circuits), this conversion needs to be done with a specific resistance value ($P = V^2/R = I^2 R$). In most noise work involving active devices, however, a standard resistance value of 1 Ω is assumed.

Integrated Noise

To make rational design choices, we need to know what the total noise variation is; this section gives us that capability. We will convert the PSD to the statistical variance (or standard deviation squared) using a definite integral across frequency.

CALCULATIONS

Using Equation 1, and the fact that the power in a frequency bin is independent of all other bins, we can add up all of the bin powers together:

EQUATION 2:   TOTAL NOISE VARIATION

$$N \approx \sum_k (PSD(f_k) \cdot \Delta f_k)$$

$$N = \int_0^\infty PSD(f)\, df$$

Where:

N = total noise power (W)

We use the summation approximation for measured noise data at discrete time points. The integral applies to continuous time noise; it is useful for deriving theoretical results.

PREFERRED EQUATIONS

In circuit analysis, the conversion to integrated noise (E_n) usually takes place with the noise voltage density; see Equation 3. E_n is the noise's standard deviation.

EQUATION 3:   INTEGRATED NOISE VOLTAGE

$$E_n = \sqrt{\int_0^\infty e^2{}_n(f)\, df}$$

Where:

$e_n(f)$ = noise voltage density (V/√Hz)

= $\sqrt{PSD(f) \cdot (1\ \Omega)}$

E_n = integrated noise voltage (V_{RMS})

= standard deviation (V_{RMS})

Noise current densities can also be converted to integrated noise (I_n):

EQUATION 4: INTEGRATED NOISE CURRENT

$$I_n = \sqrt{\int_0^\infty i^2{}_n(f)\ df}$$

Where:

$i_n(f)$ = noise current density (A/√Hz)

= $\sqrt{PSD(f)/(1\Omega)}$

I_n = integrated noise current (A_{RMS})

= standard deviation (A_{RMS})

INTERPRETATION

We need to know the probability density function in order to make informed decisions based on the integrated (RMS) noise. For the work in this application note, the noise will have a Gaussian (Normal) probability density function.

The principle noise sources within op amps, and resistors on the PCB, are Gaussian. When they are combined, they produce a total noise that is also Gaussian. Figure 2 shows the standard Gaussian probability density function (mean = 0 and standard deviation = 1) on a logarithmic y-axis.
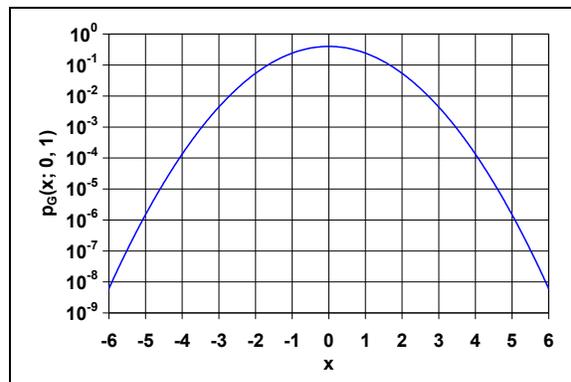

FIGURE 2: *Standard Gaussian Probability Density Function.*

Table 1 shows important points on this curve and the corresponding (two tailed) probability that the random Gaussian variable is outside of those points. This information is useful in converting RMS values (voltages or currents) to either peak or peak-to-peak values. The column label x_L is sometimes called the number of sigma from the mean.

TABLE 1: IMPORTANT TWO-TAILED PROBABILITIES

x_L	$P_G(|x| > x_L; 0, 1)$	Crest Factor (Note 1)	
		Peak (V_{PK}/V_{RMS})	Peak-to-Peak (V_{P-P}/V_{RMS})
1.64	10%	1.64	3.29
2.58	1%	2.58	5.15
3.29	0.1%	3.29	6.58
4.50	6.80×10^{-6}	4.50	9.00
6.00	1.97×10^{-9}	6.00	12.00

Note 1: Microchip's op amp data sheets use 6.6 V_{P-P}/V_{RMS} when reporting E_{ni} (usually between 0.1 Hz and 10 Hz). This is about the range of visible noise on an analog oscilloscope trace.

The integrated noise results in this application note are independent of frequency and time. They can only be used to describe noise in a global sense; correlations between the noise seen at two different time points are lost after the integration is done.

Filtered Noise

Any time we measure noise, it has been altered from its original form seen within the physical noise source. The easiest way to represent these alterations to the noise, in linear systems, is by the transfer function (in the frequency domain) from the source to the output. The resulting output noise has a different spectral shape than the source.

TRANSFER FUNCTIONS AND NOISE

It turns out [3, 4, 5] that the noise at the output of a linear operation (represented by the transfer function) is related to the input noise by the transfer function's squared magnitude; see Equation 5. This can be thought of as a result of the statistical independence between the PSD's frequency bins (see Figure 1).

EQUATION 5: OUTPUT NOISE

$$e^2_{nout} = \left| \frac{V_{OUT}}{V_{IN}} \right|^2 e^2_{ni}$$

Where:

e_{ni} = noise voltage density at V_{IN} (V/√Hz)

e_{nout} = noise voltage density at V_{OUT} (V/√Hz)

Example 1 shows the conversion of a simple transfer function to its squared magnitude. It starts as a Laplace Transform [2], it is converted to a Fourier Transform (substituting jω for s) and then converted to its squared magnitude form (a function of ω^2). It is best to do this last conversion with the transform in factored form.

EXAMPLE 1: TRANSFER FUNCTION CONVERSION EXAMPLE

Laplace Transfer Function:

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + s/\omega_P}$$

Conversion to Fourier Transfer Function:

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega/\omega_P}, \quad s \to j\omega$$

Conversion to Magnitude Squared:

$$\left|\frac{V_{OUT}}{V_{IN}}\right|^2 = \left|\frac{1}{1 + j\omega/\omega_P}\right|^2 = \frac{1}{1 + (\omega/\omega_P)^2}$$

$$= \frac{1}{1 + (f/f_P)^2}, \quad \omega \to 2\pi f$$

Where:

s	=	Laplace frequency (1/s)
	=	$\sigma + j\omega$
ω	=	Radian frequency (rad/s)
ω_P	=	Pole (rad/s)
f	=	Frequency (Hz)
f_P	=	Pole frequency (Hz)

BRICK WALL FILTERS

The transfer function that is easiest to manipulate mathematically is the brick wall filter. It has infinite attenuation (zero gain) in its stop bands, and constant gain (H_M) in its pass band; see Figure 3.

FIGURE 3: *Brick Wall Filter.*

We will use three variations of the brick wall filter (refer to Figure 3):

- Low-pass (f_L is at zero)
  - $f_L = 0 < f_H < \infty$
- Band-pass (as shown)
  - $0 < f_L < f_H < \infty$
- High-pass (f_H is at infinity)
  - $0 < f_L < \infty = f_H$

Brick wall filters are a mathematical convenience that simplifies our noise calculations.

In the physical world, however, brick wall filters would have horrible behavior. They cannot be realized with a finite number of circuit elements. Physical filters that try to approach this ideal show three basic problems: their step response exhibits Gibbs phenomenon (overshoot and ringing that decays slowly), they suffer from noise enhancement (due to high pole quality factors) and they are very difficult to implement.

> **Note:** Comments in the literature (e.g., in filter textbooks) about "ideal" brick wall filters should be viewed with skepticism.

The integrated noise voltage integrals (Equation 3 and Equation 4) are in their most simple terms when a brick wall filter is used. Equation 6 shows that, in this case, the brick wall filter's frequencies f_L and f_H become the new integration limits. The integrated current noise is treated similarly.

EQUATION 6: INTEGRATED NOISE WITH BRICK WALL FILTER

$$E_{nout} = \sqrt{\int_0^\infty e^2{}_{nout}(f)\ df}$$

$$= \sqrt{\int_0^\infty e^2{}_{ni}(f)\left|\frac{V_{OUT}}{V_{IN}}\right|^2 df}$$

$$= H_M \cdot \sqrt{\int_{f_L}^{f_H} e^2{}_{ni}(f)\ df}$$

Where:

f_L	=	Lower cutoff frequency (Hz)
f_H	=	Upper cutoff frequency (Hz)
H_M	=	Pass band gain (V/V)

See **Appendix B: "Computer Aids"** for popular circuit simulators and symbolic mathematics packages that help in these calculations.

White Noise

White noise has a PSD that is constant over frequency. It received its name from the fact that white light has an equal mixture of all visible wavelengths (or frequencies). This is a mathematical abstraction of real world noise phenomena.

A truly white noise PSD would produce an infinite integrated noise. Physically, this is not a concern because all circuits and physical materials have limited bandwidth.

We start with white noise because it is the easiest to manipulate mathematically. Other spectral shapes will be addressed in subsequent sections.

NOISE POWER BANDWIDTH

When white noise is passed through a brick wall filter (see Figure 3), the integrated noise becomes a very simple calculation. Equation 6 is simplified to:

EQUATION 7: INTEGRATED WHITE NOISE WITH BRICK WALL FILTER

$$E_{nout} = H_M e_{ni} \sqrt{f_H - f_L}$$

Where:

e_{ni}	=	Input noise voltage density (V/√Hz)
e_{nout}	=	Output noise voltage density (V/√Hz)

This equation is usually represented by what is called the Noise Power Bandwidth (NPBW). NPBW is the bandwidth (under the square root sign) that converts a white noise density into the correct integrated noise value. For the case of brick wall filters, we can use Equation 8.

EQUATION 8: INTEGRATED WHITE NOISE WITH NPBW

$$E_{nout} = H_M e_{ni} \sqrt{NPBW}$$

Where:

$NPBW = f_H - f_L,$ for brick wall filters

The high-pass filter appears to cause infinite integrated noise. In real circuits, however, the bandwidth is limited, so f_H is finite (a band-pass response).

> **Note:** NPBW applies to white noise only; other noise spectral shapes require more sophisticated formulas or computer simulations.

Circuit Noise Sources

This section discusses circuit noise sources for different circuit components and transfer functions between sources and the output.

DIODE SHOT NOISE

Diodes and bipolar transistors exhibit shot noise, which is the effect of the electrons crossing a potential barrier at random arrival times. The equivalent circuit model for a diode is shown in Figure 4.
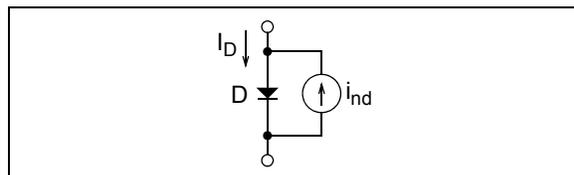

FIGURE 4: *Physically Based Noise Model for Diodes.*

The shot noise current density's magnitude depends on the diode's DC current (I_D) and the electron charge (q). It is usually modeled as white noise; see Equation 9.

EQUATION 9: DIODE SHOT NOISE

$$i_{nd} = \sqrt{2q|I_D|}$$

Where:

q	=	Electron charge
	=	1.602×10^{-19} (C)
I_D	=	Diode Current (A)

Let's look at a specific example:

EXAMPLE 2: A DIODE SHOT NOISE CALCULATION

Given:

I_D = 1 mA

Calculate:

$$i_{nd} = \sqrt{2(1.602 \times 10^{-19} C)(1\ mA)}$$
$$= 17.9\ pA/\sqrt{Hz}$$

> **Note:** All of the calculation results in this application note show more decimal places than necessary; two places are usually good enough. This is done to help the reader verify his or her calculations.

RESISTOR THERMAL NOISE

The thermal noise present in a resistor is usually modeled as white noise (for the frequencies and temperatures we are concerned with). This noise depends on the resistor's temperature, not on its DC current. Any resistive material exhibits this phenomenon, including conductors and CMOS transistors' channel.

Figure 5 shows the models for resistor thermal noise voltage and current densities. The sources are shown with a polarity for convenience in circuit analysis.
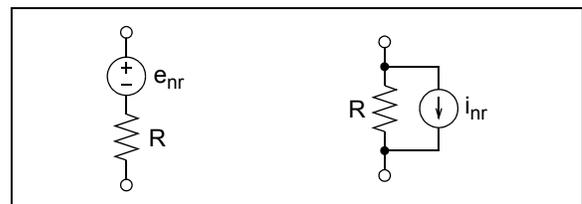

FIGURE 5: *Physically Based Noise Model for Resistors.*

The equivalent noise voltage and current spectral densities are (remember that 273.15 K = 0°C):

EQUATION 10: RESISTOR THERMAL NOISE DENSITY

$$e_{nr} = \sqrt{4kT_A R}$$

$$i_{nr} = \sqrt{4kT_A / R}$$

Where:

k = Boltzmann constant

= 1.381×10^{-23} (J/K)

T_A = Ambient temperature (K)

R = Resistance (Ω)

$4kT_A$ represents a resistor's internal power. The maximum available power to another resistor is kT_A (when they are equal). Many times the maximum available power is shown as $kT_A/2$ because physicists prefer using two-sided noise spectra.

Let's use a 1 kΩ resistor as an example.

EXAMPLE 3: A THERMAL NOISE DENSITY CALCULATION

Given:

R = 1 kΩ

T_A = 25°C = 298.15 K

Calculate the noise voltage density:

$$e_{nr} = \sqrt{4(1.381 \times 10^{-23} \, J/K)(298.15 \, K)}$$

$$= 4.06 \, nV/\sqrt{Hz}$$

Calculate the noise current density:

$$i_{nr} = \sqrt{4(1.381 \times 10^{-23} \, J/K)/(298.15 \, K)}$$

$$= 4.06 \, pA/\sqrt{Hz}$$

OP AMP NOISE

An op amp's noise is modeled with three noise sources: one for the input noise voltage density (e_{ni}) and two for the input noise current density (i_{bn} and i_{bi}). All three noise sources are physically independent, so they are statistically uncorrelated. Figure 6 shows this model; it is similar to the DC error model covered in [1].

FIGURE 6: *Physically Based Noise Model for Op Amps.*

The noise voltage source can also be placed at the other input of the op amp, with its negative pin is connected to V_I and its positive pin to V_M. This alternate connection gives the same output voltage (V_{OUT}).

For voltage feedback (VFB) op amps, both noise current sources have the same magnitude. This magnitude is shown in Microchip's op amp data sheets with the symbol i_{ni}; it has units of fA/√Hz (f stands for femto, or 10^{-15}).

For now, we will discuss the white noise part of these spectral densities. We will defer a discussion on 1/f noise until later.

The literature sometimes shows an amplifier noise model that has only one noise current source. In these cases, the second noise current's power has been combined into the noise voltage magnitude.

> **Note:** Keep in mind that op amps have two physically independent noise current sources.

For current feedback (CFB) op amps, the two noise current sources (i_{bn} and i_{bi}) are different in magnitude because the two input bias currents (I_{BN} and I_{BI}) are different in magnitude. They are produced by physically independent and statistically uncorrelated processes. CFB op amps are typically used in wide bandwidth applications (e.g., above 100 MHz).

Microchip's CMOS input op amps have a noise current density based on the input pins' ESD diode leakage current (specified as the input bias current, I_B). Table 2 gives the MCP6241 op amp's white noise current values across temperature.

TABLE 2: MCP6241 (CMOS INPUT) NOISE CURRENT DENSITY

T_A (°C)	I_B (pA)	i_{ni} (fA/√Hz)
25	1	0.57
85	20	2.5
125	1100	19

Table 3 gives the MCP616 op amp's white input noise current density across temperature. This part has a bipolar (PNP) input; the base current is the input bias current, which decreases with temperature.

TABLE 3:   MCP616 (BIPOLAR INPUT) NOISE CURRENT DENSITY

T_A (°C)	I_B (nA)	i_{ni} (fA/√Hz)
-40	-21	82
25	-15	69
85	-12	62

The input noise voltage density (e_{ni}) typically does not change much with temperature.

Note:	Noise current density (i_{ni}) usually changes significantly with temperature (T_A).
Note:	Most of the time, you can use I_B vs. T_A and the shot noise formula to calculate i_{ni} vs. T_A. One exception to this rule is op amps with input bias current cancellation circuitry.

TRANSFER FUNCTIONS

The transfer function from each noise source in a circuit to the output is needed. This may be obtained with SPICE simulations (see **Appendix B: "Computer Aids"**) or with analysis by hand. This application note emphasizes the manual approach more in order to build understanding and to derive handy design approximations.

The most convenient manual approach is circuit analysis using the Laplace frequency variable (s). Figure 7 shows a resistor, inductor and capacitor with their corresponding impedances (using s).

FIGURE 7:   *Impedance Models for Common Passive Components.*

NOISE ANALYSIS PROCESS

This section goes through the analysis process normally followed in noise design. It uses a very simple noise design problem to make this process clear.

Simple Example

The circuit shown in Figure 8 uses an op amp and a lowpass brick wall filter ($f_L = 0$). The filter's bandwidth (f_H) is 10 kHz and its gain (H_M) is 1 V/V. The op amp's input noise voltage density (e_{ni}) is 100 nV/√Hz, and its gain bandwidth product is much higher than f_H.
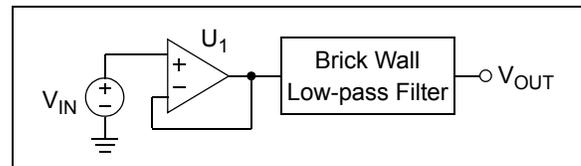

FIGURE 8:   *Op Amp Circuit.*

Figure 9 shows both the op amp noise voltage density (e_{ni}) and the output noise voltage density (e_{nout}). Notice that e_{nout} is simply e_{ni} multiplied by the low-pass brick wall's pass-band gain (H_M).
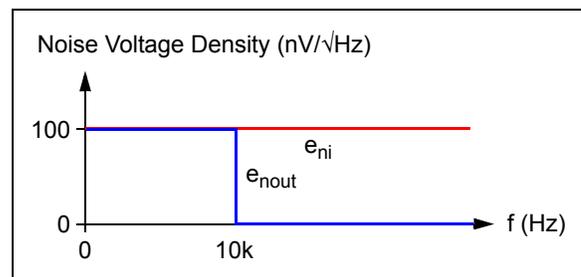

FIGURE 9:   *Noise Voltage Densities.*

The noise current densities i_{bn} and i_{bi} can be ignored in this circuit because they flow into a voltage source and the op amp output, which present zero impedance.

Now we can calculate the integrated noise at the output (E_{nout}). The result is shown in three different units (RMS, peak and peak-to-peak):

EXAMPLE 4:   AN INTEGRATED NOISE CALCULATION

$$E_{nout} = \sqrt{\int_0^\infty e^2{}_{nout}(f)\ df}$$

$$= \sqrt{\int_0^{10\ kHz} \left(100\ nV/\sqrt{Hz}\right)^2\ df}$$

$$= \left(100\ nV/\sqrt{Hz}\right)\sqrt{10\ kHz}$$

$$= 10\ \mu V_{RMS} = 33\ \mu V_{PK} = 66\ \mu V_{P\text{-}P}$$

Note:	This application note uses the crest factor 3.3 V_{PK}/V_{RMS} (or 6.6 $V_{P\text{-}P}/V_{RMS}$).

Figure 10 shows numerical simulation results of the output noise over time. E_{nout} describes the variation of this noise. This same data is plotted in histogram form in Figure 11; the curve represents the ideal Gaussian probability density function (with the same average and variation).
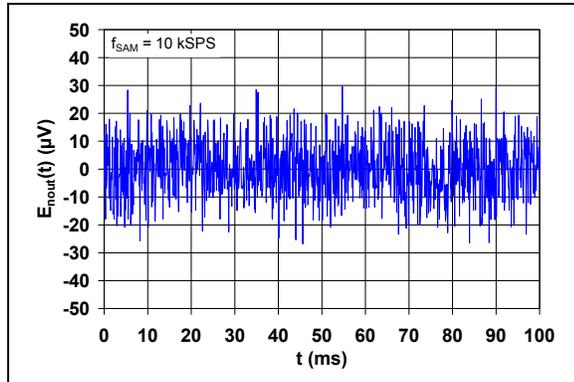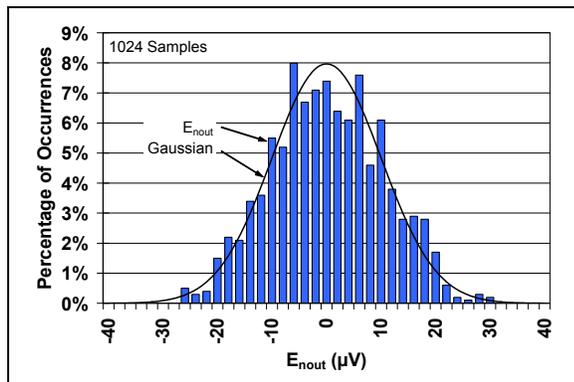

FIGURE 10: *Output Noise vs. Time.*

FIGURE 11: *Output Noise Histogram.*

Review of the Process

The basic process we have followed can be described as follows.

- Collect noise and filter information
- Convert noise at the sources to noise at the output
- Combine and integrate the output noise terms
- Evaluate impact on the output signal

FILTERED NOISE

This section covers the op amp circuits that have filters at their output. The discussion focuses on filters with real poles to develop insight and useful design formulas.

The effect that reactive circuit components have on noise is deferred to a later section. Noise generated by the filters is ignored for now.

Low-pass Filter With Single Real Pole

Figure 12 shows an op amp circuit with a low-pass filter at the output, which has a single real pole (f_P). We do not need to worry about the noise current densities because the i_{bn} and i_{bi} sources see zero impedance (like Figure 8). We will assume that the op amp BW can be neglected because f_P is much lower.

FIGURE 12: *Op Amp Circuit With Low-pass Filter.*

We need the filter's transfer function in order to calculate the output integrated noise; it needs to be in squared magnitude form (see Example 1 for the derivation of these results):

EQUATION 11: LOW-PASS TRANSFER FUNCTION

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega/\omega_P}$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right|^2 = \frac{1}{1 + (f/f_P)^2}$$

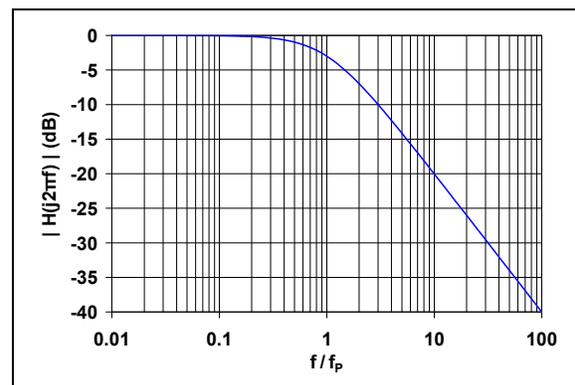Figure 13 shows the transfer function magnitude in decibels.

FIGURE 13: *Filter Magnitude Response.*

Now we can obtain the integrated noise, assuming the op amp's input noise voltage density (e_{ni}) is white:

EQUATION 12: INTEGRATED NOISE DERIVATION

$$E_{nout} = \sqrt{\int_0^\infty e^2_{nout}(f)\,df} = \sqrt{\int_0^\infty \frac{e^2_{ni}\,df}{1 + f^2/f_P^2}}$$

$$= e_{ni}\sqrt{f_P[atan(f/f_P)]_0^\infty}$$

$$= e_{ni}\sqrt{(\pi/2)\cdot f_P}$$

Thus, the NPBW for this filter is (see Equation 8):

EQUATION 13: NOISE POWER BANDWIDTH

$$NPBW = (\pi/2)\cdot f_P$$

We can always reduce the integrated output noise by reducing NPBW, but the signal response may suffer if we go too far. We need to keep the filter's -3 dB bandwidth (BW) at least as large as the desired signal BW (f_P is this filter's BW).

For low-pass filters, we can also select the BW based on the maximum allowable step response rise time [6] (this applies to any reasonable low-pass filter):

EQUATION 14: RISE TIME VS. BANDWIDTH

$$t_R \approx 0.35/BW$$

Where:

BW = Any low-pass filter's -3 dB bandwidth (Hz)

t_R = 10% to 90% Rise time (s)

Let's try a numerical example with reasonably wide bandwidth; the noise is limited by the filter's bandwidth.

EXAMPLE 5: AN INTEGRATED NOISE CALCULATION

Filter Specifications:

f_P = BW = 10 kHz

Gain = 1 V/V

Op Amp Specifications:

e_{ni} = 100 nV/√Hz

BW = 1 MHz

Filter Rise Time:

$t_R \approx 35\ \mu s$

Integrated Noise Calculations:

f_P << Op amp's bandwidth

$$NPBW = (\pi/2)\cdot(10\ kHz) = 15.8\ kHz$$

$$E_{nout} = (100\ nV/\sqrt{Hz})\cdot\sqrt{(15.8\ kHz)}$$

$$= 12.6\ \mu V_{RMS} = 41.4\ \mu V_{PK} = 82.9\ \mu V_{P-P}$$

Low-pass Filter With Two Real Poles

The low-pass filter in Figure 14 has two real poles (f_{P1} and f_{P2}). We do not need to worry about the noise current densities because the i_{bn} and i_{bi} sources see zero impedance (like Figure 8). We assume that f_{P1} and f_{P2} are much lower than the op amp BW, so the op amp BW can be neglected.
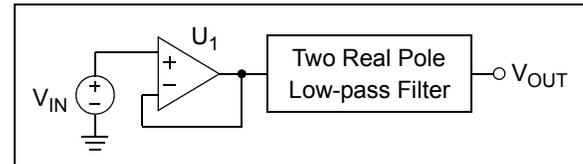

FIGURE 14: *Op Amp Circuit With Low-pass Filter.*

The filter's transfer function and the magnitude squared transfer function (a function of ω^2), in factored form, are:

EQUATION 15: LOW-PASS TRANSFER FUNCTION

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega/\omega_{P1}}\cdot\frac{1}{1 + j\omega/\omega_{P2}}$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right|^2 = \frac{1}{1 + (f/f_{P1})^2}\cdot\frac{1}{1 + (f/f_{P2})^2}$$

Where:

f_{P1} = First pole frequency (Hz)

f_{P2} = Second pole frequency (Hz)

Figure 15 shows the transfer function magnitude in decibels for the specific case where f_{P2} is double f_{P1}.
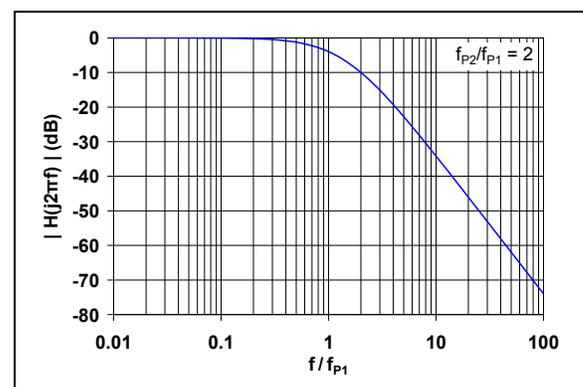

FIGURE 15: *Filter Magnitude Response.*

We can follow the same process as before to calculate NPBW.

EQUATION 16: NPBW

$$NPBW = \left(\frac{\pi}{2}\right) \Big/ \left(\frac{1}{f_{P1}} + \frac{1}{f_{P2}}\right)$$

As before, NPBW and BW are similar and BW can be traded-off with rise time (see Equation 14).

EQUATION 17: BW

$$BW = \frac{f_{P1}}{\sqrt{X + \sqrt{1 + X^2}}}, \quad X = 1 + \frac{1}{2} \cdot \left(\frac{f_{P1}}{f_{P2}}\right)^2$$

Where:

$$f_{P1} \leq f_{P2}$$

Let's go through a numerical example where the op amp's bandwidth can be neglected.

EXAMPLE 6: AN INTEGRATED NOISE CALCULATION

Filter Specifications:

f_{P1} = 13.4 kHz

f_{P2} = 26.8 kHz

Gain = 1 V/V

Op Amp Specifications:

e_{ni} = 100 nV/√Hz

BW = 1 MHz

Filter Bandwidth and Rise Time:

$BW = 9.98\ kHz$

$t_R \approx 35\ \mu s$

Integrated Noise Calculations:

f_{P2} << Op amp's bandwidth

$NPBW = 14.0\ kHz$

$E_{nout} = (100\ nV/\sqrt{Hz}) \cdot \sqrt{(14.0\ kHz)}$

$= 11.8\ \mu V_{RMS} = 39.0\ \mu V_{PK} = 78.1\ \mu V_{P\text{-}P}$

Let's redo this example with equal poles at 15.5 kHz.

EXAMPLE 7: AN INTEGRATED NOISE CALCULATION

Change in Filter Specifications:

$f_{P1} = f_{P2}$ = 15.5 kHz

Filter Bandwidth and Rise Time:

$BW = 9.98\ kHz$

$t_R \approx 35\ \mu s$

Integrated Noise Calculations:

f_{P2} << Op amp's bandwidth

$NPBW = 12.2\ kHz$

$E_{nout} = (100\ nV/\sqrt{Hz}) \cdot \sqrt{(12.2\ kHz)}$

$= 11.0\ \mu V_{RMS} = 36.4\ \mu V_{PK} = 72.9\ \mu V_{P\text{-}P}$

High-pass Filter With Single Real Pole

Figure 16 shows an op amp circuit with a high-pass filter with a single real pole (f_P). We do not need to worry about the noise current densities because the i_{bn} and i_{bi} sources see zero impedance (like Figure 8). For practical circuits, there needs to be a low-pass filter at a frequency much higher than f_P (at f_H); the integrated noise would be infinite otherwise. If nothing else, the op amp BW may be used to limit the NPBW.
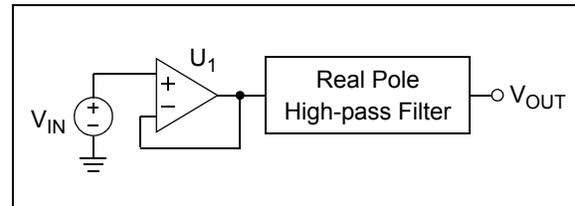

FIGURE 16: *Op Amp Circuit With High-pass Filter.*

The filter's transfer function and the magnitude squared transfer function (a function of ω^2), in factored form, are:

EQUATION 18: HIGH-PASS TRANSFER FUNCTION

$$\frac{V_{OUT}}{V_{IN}} = \frac{j\omega/\omega_P}{1 + j\omega/\omega_P}, \quad \omega < \omega_H$$

$$= 0, \quad \omega \geq \omega_H$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right|^2 = \frac{(f/f_P)^2}{1 + (f/f_P)^2}, \quad f < f_H$$

$$= 0, \quad f \geq f_H$$

Where:

f_P = Pole frequency (Hz)

f_H = Low-pass NPBW (Hz)

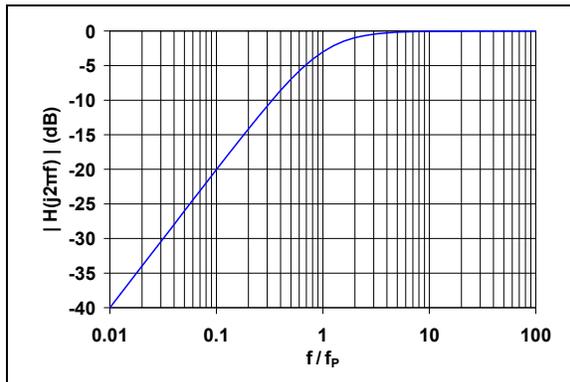Figure 17 shows the transfer function magnitude in decibels (f_H is not shown).

FIGURE 17: *Filter Magnitude Response.*

We can follow the same process as before to calculate NPBW (f_H acts like the upper integration limit in the integrated noise equation).

EQUATION 19: NPBW

$$NPBW = f_H - (\pi/2) \cdot f_P$$

Where:

$$f_P \quad << \quad f_H$$

Let do a numerical example with the op amp bandwidth much higher than the filter pole (this is very common).

EXAMPLE 8: AN INTEGRATED NOISE CALCULATION

Filter Specifications:

f_P = 10 kHz

f_H = Op amp's NPBW

Gain = 1 V/V

Op Amp Specifications:

e_{ni} = 100 nV/√Hz

BW = 1 MHz

$NPBW \approx (\pi/2) \cdot BW = 1.57\ MHz$

Integrated Noise Calculations:

f_P << Op amp's bandwidth

$NPBW = (1.57\ MHz) - (15.8\ kHz) = 1.55\ MHz$

$E_{nout} = (100\ nV/\sqrt{Hz}) \cdot \sqrt{(1.55\ MHz)}$

$= 124\ \mu V_{RMS} = 411\ \mu V_{PK} = 822\ \mu V_{P-P}$

Note:	A high-pass filter's NPBW has little effect on the integrated noise, unless f_H is near f_P (but that would be a band-pass filter).

Band-pass Filter With Two Real Poles

Figure 18 shows an op amp circuit with a band-pass filter with two real poles (highpass pole f_{P1} and low-pass pole f_{P2}). We do not need to worry about the noise current densities because the i_{bn} and i_{bi} sources see zero impedance (like Figure 8). The op amp BW is neglected because we assume that it is much higher than f_{P1} and f_{P2}.
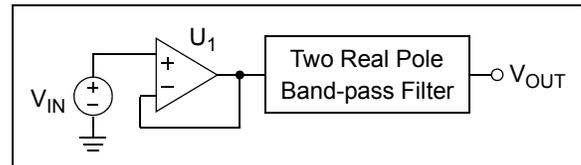

FIGURE 18: *Op Amp Circuit With Band-pass Filter.*

The filter's transfer function and the magnitude squared transfer function (a function of ω^2), in factored form, are:

EQUATION 20: BAND-PASS TRANSFER FUNCTION

$$\frac{V_{OUT}}{V_{IN}} = \frac{j\omega/\omega_{P1}}{1 + j\omega/\omega_{P1}} \cdot \frac{1}{1 + j\omega/\omega_{P2}}$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right|^2 = \frac{(f/f_{P1})^2}{1 + (f/f_{P1})^2} \cdot \frac{1}{1 + (f/f_{P2})^2}$$

Where:

f_{P1} = High-pass pole frequency (Hz)

f_{P2} = Low-pass pole frequency (Hz)

Figure 19 shows the transfer function magnitude in decibels, with f_{P2} = 100 f_{P1}.
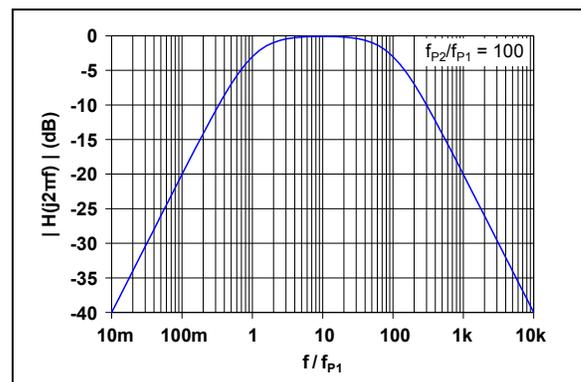

FIGURE 19: *Filter Magnitude Response.*

Using a symbolic solver to derive NPBW is a big help.

EQUATION 21: NPBW

$$NPBW = (\pi/2) \cdot f_{P2} \cdot \frac{1}{1 + f_{P1}/f_{P2}}$$

Let's do another numerical example.

EXAMPLE 9: AN INTEGRATED NOISE CALCULATION

Filter Specifications:

f_{P1} = 100 Hz

f_{P2} = 10 kHz

Gain = 1 V/V

Op Amp Specifications:

e_{ni} = 100 nV/√Hz

BW = 1 MHz

Integrated Noise Calculations:

f_{P2} << Op amp's bandwidth

$NPBW = (15.7 \, kHz)/1.01 = 15.5 \, kHz$

$E_{nout} = (100 \, nV/\sqrt{Hz}) \cdot \sqrt{(15.5 \, kHz)}$

$= 12.5 \, \mu V_{RMS} = 41.2 \, \mu V_{PK} = 82.3 \, \mu V_{P-P}$

Comments on Other Filters

This section discusses other filters and how they affect the output integrated noise. It gives a very simple approximation to NPBW when the filter order is greater than n = 1. It then discusses noise generated interal to a filter.

SOME SIMPLE LOW-PASS FILTERS

Table 4 shows the NPBW to BW ratio for some low-pass filters up to order 5.

TABLE 4: NPBW FOR SOME LOW-PASS FILTERS

Low-pass Filter Type	NPBW / BW				
	n = 1	n = 2	n = 3	n = 4	n = 5
Identical Real Poles	1.571	1.220	1.155	1.128	1.114
Bessel	1.571	1.153	1.071	1.046	1.038
Butterworth	1.571	1.111	1.047	1.026	1.017

Note: The -3 dB bandwidth is a rough estimate of NPBW for almost all filters (the main exception is when n = 1).

FILTERS WITH GREATER SELECTIVITY

There are other filters with a sharper transition region, when n > 1, such as: Chebyshev, Inverse Chebyshev and Elliptic filters. Their NPBW to BW ratios are closer to 1 because they have a smaller transition region (between pass-band and stop-band). This smaller transition region reduces the integrated noise at the output. Their step response, however, tends to have more ringing and slower decay.

Again, NPBW can be approximated with the -3 dB bandwidth. More exact results can be obtained with simulations (see **Appendix B: "Computer Aids"**).

NOISE INTERNAL TO FILTERS

As will be shown later (see Figure 25), active filters may produce much more noise than first expected. The op amps inside the filter produce a noise voltage density at the filter's output that has a wider bandwidth than the filter; it may be as wide as the op amp band-widths. The resistors and op amp noise contributions tend to show a peak at the edges of the filter passband (noise enhancement), which increases the integrated output noise.

MULTIPLE NOISE SOURCES

This section covers two approaches to combining multiple noise sources into one output integrated noise result. This knowledge is applied to a simple R-C low-pass filter and a non-inverting gain circuit.

Combining Noise Outputs

When we combine noise results, at the output, we take advantage of the statistical independence of:

• PSD noise in separate frequency bins
• Physically independent noise sources

This independence simplifies our work, since we do not need to worry about correlations.

We can integrate the output noise densities one at a time, then combine the results using a Sum of Squares approach (see Equation 1). We can also combine all of the noise densities using a Sum of Squares approach first, then integrate the resulting noise density.

FIGURE 20: *Approaches to Combining Output Noise Terms.*

Each approach has its advantages. Integrating first helps determine which noise source dominates; it is handy for hand designs. Finding the output noise density first helps to adjust frequency shaping elements in the design; it is easier with computer simulations.

The next section (**"R-C Low-pass Filter"**) demonstrates the approach on the left of Figure 20. The section following that one (**"Non-inverting Gain Circuit"**) demonstrates the approach on the right of Figure 20.

R-C Low-pass Filter

Figure 21 shows a circuit with a R-C low-pass filter with a real pole (f_P). We do not need to worry about the noise current densities because the i_{bn} and i_{bi} sources see zero impedance (like Figure 8). We will assume that the op amp BW can be neglected because f_P is much lower.
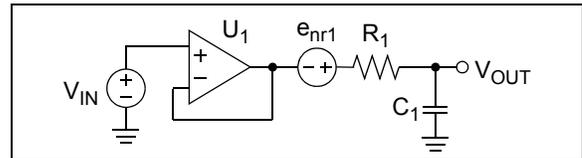

FIGURE 21: *Circuit With R-C Low-pass Filter.*

We will integrate the noise densities first because this will give us important insight into this R-C low-pass filter. This circuit is like the one we already saw in Figure 12, but we have added R_1's thermal noise.

The filter's transfer function and the magnitude squared transfer function (a function of ω^2), in factored form, are in Equation 22 (Figure 13 shows the transfer function magnitude in decibels).

EQUATION 22: R-C LOW-PASS FILTER TRANSFER FUNCTION

$$\frac{V_{OUT}}{V_{IN}} = \frac{1/(sC_1)}{R_1 + 1/(sC_1)} = \frac{1}{1 + sR_1C_1} \rightarrow \frac{1}{1 + j\omega/\omega_P}$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right|^2 = \left|\frac{V_{OUT}}{e_{nr1}}\right|^2 = \frac{1}{1 + (f/f_P)^2}$$

Where:

f_P = R-C filter's pole frequency (Hz)

$= 1/(2\pi R_1 C_1)$

ω_P = $2\pi f_P$

We can follow the same process as before to calculate NPBW. The trade-offs between NPBW (or BW) and t_R shown in Equation 14 apply to this filter.

EQUATION 23: NPBW

$$NPBW = (\pi/2) \cdot f_P$$

The integrated noise becomes:

EQUATION 24: INTEGRATED NOISE

$$E_{noU1} = e_{ni} \cdot \sqrt{NPBW}$$

$$E_{noR1} = e_{nr1} \cdot \sqrt{NPBW}$$

$$= \sqrt{4kT_A R_1} \cdot \sqrt{(\pi/2)/(2\pi R_1 C_1)}$$

$$= \sqrt{kT_A/C_1}$$

$$E_{nout} = \sqrt{E_{noU1}^2 + E_{noRC}^2}$$

Where:

E_{noU1}	=	U_1's output integrated noise (V_{RMS})
E_{noR1}	=	R_1's output integrated noise (V_{RMS})
E_{nout}	=	Total output integrated noise (V_{RMS})

The last expression shown for E_{noR1} ($sqrt(kT_A/C_1)$) is popularly called "kT on C noise" (referring to the PSD inside the square root). This result applies only to this particular case (integrated thermal noise at the output of an R-C lowpass filter).

> **Note:** Do not let this equation mislead you; R_1 generates the thermal noise, not C_1.

Let's do a numerical example where the op amp and the filter resistor both contribute to the noise.

EXAMPLE 10: AN INTEGRATED NOISE CALCULATION

Ambient Temperature:

$T_A = 25°C = 298.15\ K$

Filter Specifications:

$R_1 = 10\ k\Omega$
$C_1 = 1.5\ nF$
Gain = 1 V/V

Op Amp Specifications:

$e_{ni} = 100\ nV/\sqrt{Hz}$

BW = 1 MHz

Filter Pole, Bandwidth and Rise Time:

$f_P = BW = 10.6\ kHz$

$t_R \approx 33\ \mu s$

Integrated Noise Calculations:

$f_P << $ *Op amp's bandwidth*

$NPBW = (\pi/2) \cdot (10.6\ kHz) = 16.7\ kHz$

$E_{noU1} = (100\ nV/\sqrt{Hz}) \cdot \sqrt{(16.7\ kHz)}$

$\qquad = 12.9\ \mu V_{RMS}$

$e_{nr1} = 12.8\ nV/\sqrt{Hz}$

$E_{noRC} = (12.8\ nV/\sqrt{Hz}) \cdot \sqrt{(16.7\ kHz)}$

$\qquad = 1.66\ \mu V_{RMS}$

$E_{nout} = 13.0\ \mu V_{RMS} = 43.0\ \mu V_{PK} = 85.9\ \mu V_{P-P}$

Non-inverting Gain Circuit

Figure 22 is a complete model for a non-inverting gain circuit. R_1 and R_3 use the series noise voltage density sources because their transfer function to V_{OUT} is simpler in that form. R_2 uses the shunt noise current density source because we can use the same transfer function to V_{OUT} that i_{bi} uses; this reduces our work.
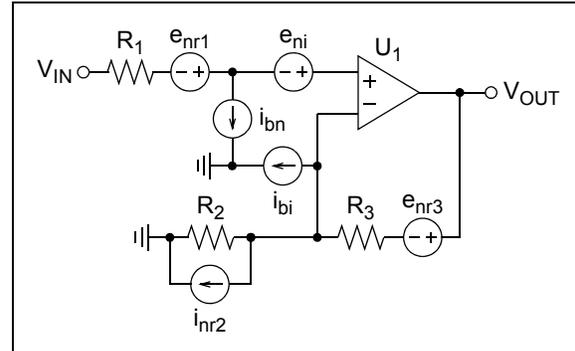

FIGURE 22: *Non-inverting gain Amplifier, with multiple noise sources.*

ANALYSIS WITH CONSTANT GAINS

We will combine the noise densities first to obtain the output noise density (e_{nout}). In this case, because we have no reactive elements in the circuit, it will be a simple matter to integrate e_{nout} by hand to produce E_{nout}.

We will start with all of the transfer functions from each source to V_{OUT} (see reference [1]). The gains will be assumed constant for now; we will deal with frequency shaping later on.

EQUATION 25: TRANSFER FUNCTIONS

$$\frac{V_{OUT}}{V_{IN}} = \frac{V_{OUT}}{e_{nr1}} = \frac{V_{OUT}}{e_{ni}} = G_N$$

$$\frac{V_{OUT}}{e_{nr3}} = 1$$

$$\frac{V_{OUT}}{i_{bn}} = -R_1 G_N$$

$$\frac{V_{OUT}}{i_{bi}} = \frac{V_{OUT}}{i_{nr2}} = R_3$$

Where:

G_N	=	Noise Gain (V/V)
	=	$1 + R_3/R_2$

Note that noise gain (G_N) is from the non-inverting input to V_{OUT}, when the op amp is in a closed-loop condition, and when other (external) energy sources are zero.

> **Note:** The concept of noise gain is central to understanding op amp behavior. It simplifies op amp bandwidth and stability analyses.

The magnitude squared transfer functions are simply the squares of the constant terms in Equation 25. We will now combine these noise densities into one equation for the output noise density (using a sum of squares approach):

EQUATION 26: COMBINING NOISE DENSITIES

$$e_{nout}^2 = G_N^2(e_{nr1}^2 + e_{ni}^2 + i_{bn}^2 R_1^2) + e_{nr3}^2 + (i_{bi}^2 + i_{nr2}^2)R_3^2$$
$$= G_N^2(4kT_A R_1 + e_{ni}^2 + i_{bn}^2 R_1^2) + 4kT_A R_3$$
$$+ (i_{bi}^2 + 4kT_A/R_2)R_3^2$$

While this equation is sufficient to calculate E_{nout}, converting to an input referred form gives more insight to the designer. Dividing both sides by G_N^2, substituting $1 + R_3/R_2$ for G_N, and simplifying, gives:

EQUATION 27: OP AMP NOISE EQUATION

$$e_{nout}^2/G_N^2 = e_{ni}^2 + i_{bn}^2 R_1^2 + i_{bi}^2(R_2\|R_3)^2$$
$$+ 4kT_A(R_1 + (R_2//R_3))$$

This shows that the output noise density has a very simple relationship to the resistances seen by the inputs (R_1 and ($R_2\|R_3$)).

Note:	Equation 27 also applies to inverting amplifiers (i.e., R_2 is driven by V_{IN} and R_1 is grounded).

ANALYSIS WITH LIMITED BANDWIDTH

To produce a finite output integrated noise, we need a filter that limits the NPBW. This filter can be implemented with the op amp, by reactive elements in the circuit (e.g., capacitors) or by a filter after the op amp.

We can use the op amp's BW to set NPBW. The response can be approximated with a single real pole for hand calculations. The Gain Bandwidth Product (GBWP) specification in VFB op amp data sheets gives:

EQUATION 28: NPBW SET BY OP AMP'S BANDWIDTH

$$BW \approx GBWP/G_N$$
$$NPBW \approx (\pi/2) \cdot BW$$

Where:

GBWP	=	Gain Bandwidth Product (Hz)
BW	=	Bandwidth (Hz)
NPBW	=	Noise Power Bandwidth (Hz)

Note:	CFB op amp data sheets specify BW instead of GBWP.

Reactive elements in the circuit will require a more detailed analysis because each noise source may have a different frequency shape.

The following example has all of the noise sources at about the same magnitude.

EXAMPLE 11: AN INTEGRATED NOISE CALCULATION

Ambient Temperature:
  $T_A = 25°C = 298.15\ K$

Circuit Specifications:
  $R_1 = 100\ k\Omega$
  $R_2 = R_3 = 200\ k\Omega$

Op Amp Specifications:
  $e_{ni} = 100\ nV/\sqrt{Hz}$
  $i_{ni} = 1\ pA/\sqrt{Hz}$
  $GBWP = 100\ kHz$

Preliminary Calculations:
  $G_N = 2.00\ V/V$
  $(R_2\|R_3) = 100\ k\Omega$

Input Noise Densities:
  $e_{ni}^2 = (100\ nV/\sqrt{Hz})^2$
  $i_{bn}^2 R_1^2 = (100\ nV/\sqrt{Hz})^2$
  $i_{bi}^2(R_2//R_3)^2 = (100\ nV/\sqrt{Hz})^2$
  $4kT_A(R_1 + (R_2//R_3)) = (57.4\ nV/\sqrt{Hz})^2$

Output Noise Density:
  $e_{nout}^2/G_N^2 = (182\ nV/\sqrt{Hz})^2$
  $e_{nout} = 365\ nV/\sqrt{Hz}$

Integrated Output Noise:
  $GBWP = 100\ kHz$
  $BW \approx 50\ kHz$
  $NPBW \approx 78.6\ kHz$
  $E_{nout} \approx (365\ nV/\sqrt{Hz}) \cdot \sqrt{(78.6\ kHz)}$
  $\approx 102\ \mu V_{RMS} = 338\ \mu V_{PK} = 675\ \mu V_{P-P}$

Simulated Examples

This section covers two filter designs. It uses SPICE simulations to quickly obtain numerical results. The first design demonstrates potential issues with op amp circuits that need good noise performance. The second design improves the noise performance dramatically using simple changes.

SECOND ORDER FILTER

Figure 23 shows a second order Butterworth filter with a bandwidth of 1 kHz. It uses the MCP616 for the op amp; we will assume that it has no 1/f noise for now.

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The resistor R_3 balances the resistances seen by the op amp inputs, which minimizes the output offset due to input bias currents [1]. It uses the Sallen-Key topology.
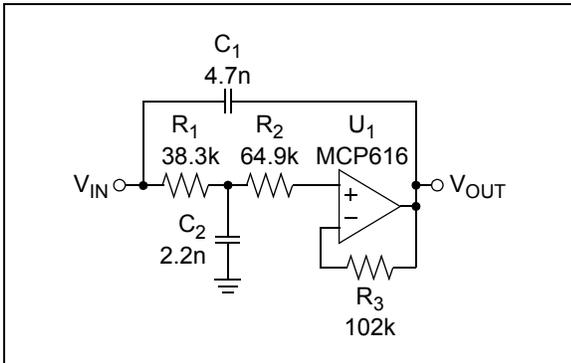

FIGURE 23: *Butterworth Lowpass Filter.*

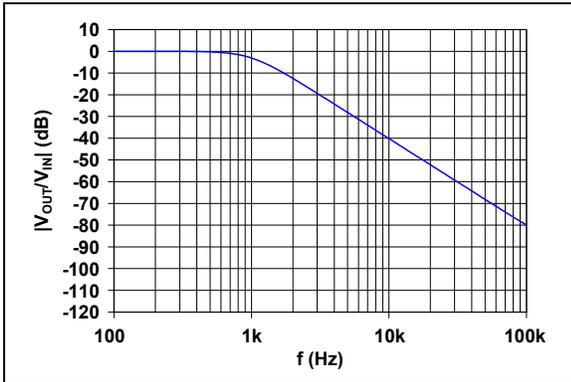Figure 24 shows the simulated transfer function for Figure 23.

FIGURE 24: *Filter Transfer Function.*

Figure 25 shows the output noise voltage densities; the labels indicate the source of a particular output density. e_{nr1}, e_{nr2} and e_{nr3} represent R_1, R_2 and R_3's thermal noise, while e_{ni}, i_{bn} and i_{bi} represent the op amp's noise sources. The combined output noise density is labeled "total."

FIGURE 25: *Output Noise Densities.*

The hump in the noise curves, seen at 1 kHz, is caused by the feedback action of the filter. The noise due to R_3 and R_2 is significant compared to the noise due to e_{ni} (the op amp's input noise voltage density).

THIRD ORDER FILTER

There are some obvious improvements we should make to this filter. Reducing the resistor values will reduce the thermal noise densities. Adding a filter at the output will significantly reduce the integrated noise at the output.

The circuit in Figure 26 is the result of making these improvements. The resistors are about four times smaller; this reduction was limited to avoid output loading concerns. The filter design was changed to a 3rd order Butterworth to take maximum advantage of the additional filter stage (R_4 and C_4).
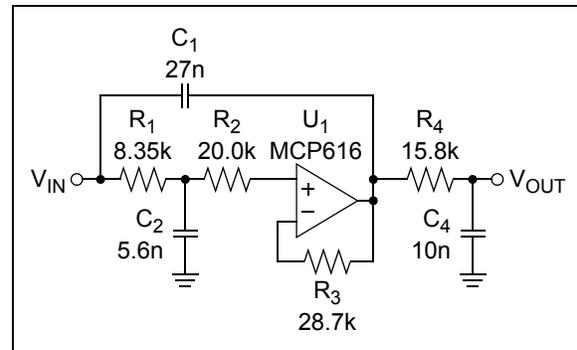

FIGURE 26: *Improved Butterworth Low-pass Filter.*

A buffer placed after R_4 and C_4 would have a wide NPBW, so its noise contribution would be significant. For this reason, the output has no output buffer.

It is possible to reduce R_3's noise contribution more by adding a capacitor (C_3, which isn't shown) in parallel to R_3. SPICE simulations will help determine if the reduction in noise is worth the additional cost.

Figure 27 shows the simulated transfer function for Figure 26; notice the improved attenuation in the stopband compared to that shown previously (see Figure 24).
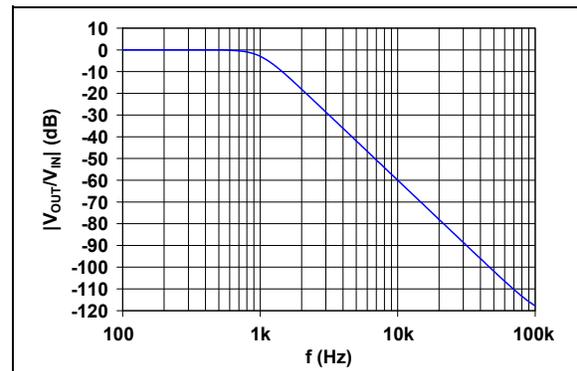

FIGURE 27: *Filter Transfer Function.*

Figure 28 shows the output noise voltage densities for Figure 26; the labels indicate the source of a particular output density. e_{nr1}, e_{nr2}, e_{nr3} and e_{nr4} represent R_1, R_2, R_3 and R_4's thermal noise, while e_{ni}, i_{bn} and i_{bi} represent the op amp's noise sources. The combined output noise density is labeled "total".
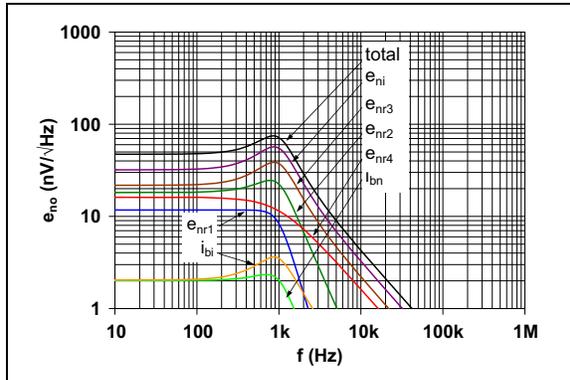

FIGURE 28: *Output Noise Densities.*

Comparing Figure 25 to Figure 28 shows that we have been successful in reducing the low frequency (i.e., below 200 Hz) output noise density. We have also reduced the overall NPBW significantly.

Table 5 compares the integrated output noise for these two designs. It summarizes the information found in Figure 25 and Figure 28 in convenient form.

TABLE 5:   COMPARISON OF DESIGNS

Noise Source		E_{no} (μV_{P-P})	
		2nd Order	3rd Order
Thermal	R_1	5.4	2.5
	R_2	10.6	5.4
	R_3	154.4	9.1
	R_4	—	4.2
Op Amp	e_{ni}	120.6	13.4
	i_{bn}	1.9	0.5
	i_{bi}	26.9	0.9
	Total	198.1	17.8

FLICKER NOISE

Flicker noise (also called 1/f noise or pink noise) can be important in low frequency applications (e.g., below 1 kHz). This noise increases the output variation above what the white noise predictions give.

> **Note:** Auto-zeroed op amps have such low 1/f noise that it can be neglected.

1/f noise is caused by defects, at the atomic level, in semiconductor and resistive devices. These defects affect the DC current flowing through these devices. With many defects operating simultaneously, each with a different time constant, 1/f noise typically results.

Components with high 1/f noise include carbon resistors and semiconductor devices (diodes and transistors). All conductors, however, exhibit 1/f noise at some level.

This section discusses 1/f noise, its impact on output variability and how to find relevant information in data sheets. A low frequency design example illustrates an approach to these designs.

1/f Noise Basics

1/f noise derives its name from its PSD shape (with units of V_{RMS}^2/Hz). The noise power increases at low frequencies as the reciprocal of frequency:

EQUATION 29:   1/F NOISE

$$e_{nf}^2(f) = e_{nf}^2(1\ Hz)/f$$

Where:

$e_{nf}(f)$ = 1/f noise voltage density at the frequency f (nV/√Hz)

> **Note:** The noise voltage density (e_{nf}) varies as 1/√f (10 dB/decade).

Notice how the 1/f noise was specified at one frequency point in Equation 29 (at 1 Hz); this is for convenience in our later work.

A DC current needs to flow for 1/f noise to be present. For instance, the PSpice diode noise model uses the following equation:

EQUATION 30:   DIODE 1/F NOISE

$$i_{nd}^2(f) = KF \cdot |I_D|^{AF}/f$$

Where:

$i_{nf}(f)$ = Diode's 1/f noise current density at the frequency f (A/√Hz)

KF = PSpice noise parameter (A^{2-AF}); default is 0 (usually around 10^{-15})

AF = PSpice noise exponent; default is 1

1/f noise is sometimes specified with a corner frequency. This happens when a noise source has both white and 1/f noise. The corner frequency occurs where the white noise density equals the 1/f noise density; see Figure 29. As we will see later, the combination of these two noise types produces a smooth bend in the region of f_{corner}, not the sharp corner depicted here.
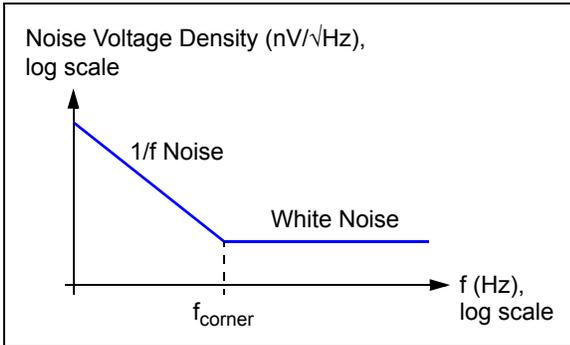

FIGURE 29: *Conceptual Diagram of the Corner Frequency.*

With the white noise density and the corner frequency, it is easy to calculate the 1/f noise voltage density (e_{nf} (1 Hz)):

EQUATION 31: CONVERSION FROM CORNER FREQUENCY

$$e_{nf}(1\ Hz) = e_{nw}\sqrt{f_{corner}/(1\ Hz)}$$

Where:

e_{nw} = White noise voltage density (nV/√Hz)

f_{corner} = corner frequency (Hz)

Figure 30 plots 1/f noise data (from bench evaluation work) that shows typical 1/f noise behavior. The data was adjusted to have zero mean and was sampled at one sample per second (1 SPS). The local average wanders over time (compare to the white noise shown in Figure 10).

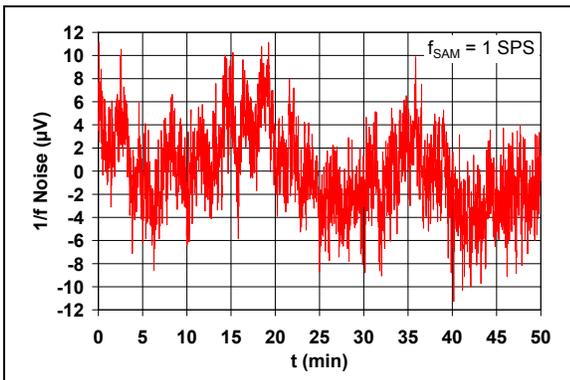**Note:** The local average of 1/f noise wanders enough to be a concern in applications.

FIGURE 30: *1/f Noise vs. Time.*

Figure 31 shows a histogram of the same noise data. The curve is the ideal Gaussian distribution with the same mean (0 µV) and standard deviation (3.55 µV).
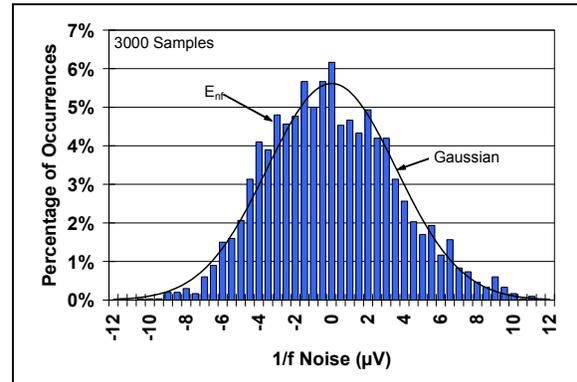

FIGURE 31: *1/f Noise Histogram.*

The first 2048 data points were converted to the noise density plot in Figure 32 (the blue curve) using a FFT routine. The red curve is the best fit 1/f noise curve (it has the same integrated noise power).
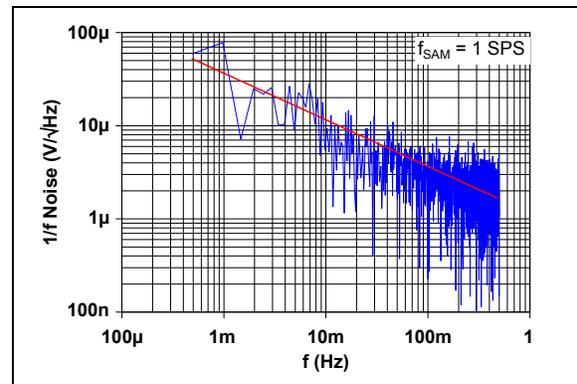

FIGURE 32: *1/f Noise; FFT (first 2048 points).*

Integrated 1/f Noise

In order to keep this analysis simple, we'll use a band-pass brick wall filter with cutoff frequencies f_L and f_H (see Figure 3). This gives:

EQUATION 32: INTEGRATED 1/F NOISE

$$E_{nf}^2 = \int_{f_L}^{f_H} e_{nf}^2(f)\ df = \int_{f_L}^{f_H} \frac{e_{nf}^2(1\ Hz)}{f}\ df$$
$$= e_{nf}^2(1\ Hz) \cdot ln(f_H/f_L)$$
$$E_{nf} = e_{nf}(1\ Hz) \cdot \sqrt{ln(f_H/f_L)}$$

In other words, the integrated power (statistical variance) is proportional to the number of decades (or octaves) encompassed by the brick wall filter.

Table 6 shows the growth that 1/f noise would exhibit with different ratios of f_H to f_L. Mathematically, E_{nf} has unbounded growth as f approaches zero. Practically speaking, however, that growth is so slow that it does not affect most applications. The numerical values are based on the data shown in Figure 32.

TABLE 6: GROWTH OF 1/F NOISE (NOTE 1, NOTE 2)

f_H/f_L	No. Decades	E_{nf} (μV_{P-P})	$1/f_L$
1.259	0.10	3.7	0.13 s
10^1	1	11.6	1.00 s
10^2	2	16.4	10 s
10^3	3	20.1	100 s
10^4	4	23.2	1000 s
10^5	5	26.0	2.78 hr
10^6	6	28.5	27.8 hr
10^7	7	30.7	11.6 day
10^8	8	32.9	116 day
10^9	9	34.9	3.17 year
3.16×10^9	9.50	35.8	10.0 year

Note 1: These numbers are based on f_H = 10 Hz and e_{nf}(1 Hz) = 1160 nV/√Hz.

2: The last entry was limited to a reasonable design lifetime for a PCB circuit.

Note: Changing the band-pass filter's ratio of f_H to f_L has little impact on 1/f noise variability when f_H/f_L > 10 and f_H << f_{corner}.

Information in Data Sheets

Table 7 shows the noise specifications in the MCP616/7/8/9 Data Sheet. This op amp family has a bipolar (PNP) input, so the noise current is higher than the CMOS input op amps.

TABLE 7: MCP616/7/8/9 NOISE SPECIFICATIONS

Parameters	Sym	Typ	Units	Conditions
Noise				
Input Noise Voltage	E_{ni}	2.2	μV_{P-P}	f = 0.1 to 10 Hz
Input Noise Voltage Density	e_{ni}	32	nV/√Hz	f = 1 kHz
Input Noise Current Density	i_{ni}	70	fA/√Hz	f = 1 kHz

The Input Noise Voltage (E_{ni}) is the integrated noise voltage between 0.1 Hz and 10 Hz, with units of (μV_{P-P}). It helps select an op amp for low frequency work. Typically, it is dominated by 1/f noise; auto-zeroed op amps are the main exception to this rule.

The Input Noise Voltage Density (e_{ni}) is usually given at a frequency where the white noise dominates (1 kHz in this case). This specification helps select an op amp for high frequency work.

The Input Noise Current Density (i_{ni}) is usually given at a frequency where the white noise dominates (1 kHz in this case). This specification helps select an op amp where the resistances are high. Remember that this curve describes both input noise current sources, which are statistically independent.

Figure 33 shows the noise density plot in the MCP616/7/8/9 Data Sheet. The noise specifications describe this data. Note that our CMOS input op amps do not show i_{ni} in this plot because it is low enough to not affect most designs.
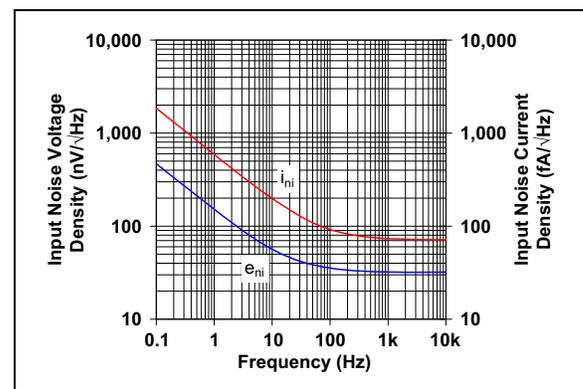

FIGURE 33: *MCP616/7/8/9 Input Noise Voltage Density Plot.*

Refer to Table 2 and Table 3 for examples of how the white noise portion of i_{ni} changes with temperature.

Design Example

This design example is a simple modification to the filter shown in Figure 26. The goal is to show a low frequency circuit that is dominated by 1/f noise. To obtain a cut-off frequency of 100 Hz, the capacitors have all been increased by a factor of 10. Figure 34 shows the result; this is still a 3rd order Butterworth filter.
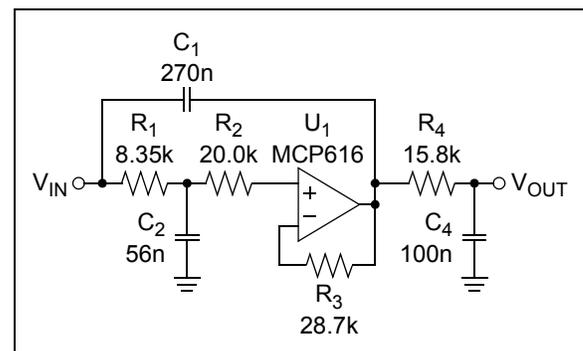

FIGURE 34: *Butterworth Low-pass Filter.*

AN1228

A buffer placed after R_4 and C_4 would have a wide NPBW, so its noise contribution would be significant. For this reason, the output has no output buffer.

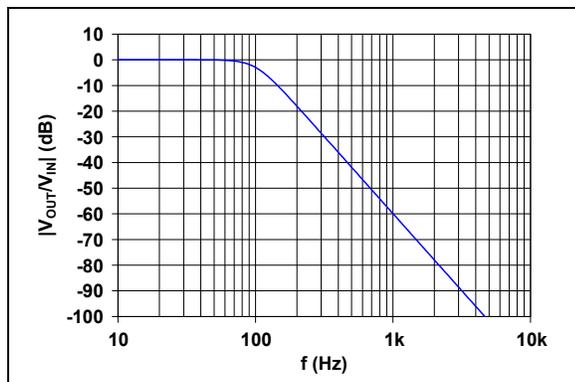Figure 35 shows the simulated transfer function (compare to Figure 27).

FIGURE 35:   *Filter Transfer Function.*

Figure 36 shows the output noise voltage densities; the labels indicate the source of a particular output density. e_{nr1}, e_{nr2}, e_{nr3} and e_{nr4} represent R_1, R_2, R_3 and R_4's thermal noise, while e_{ni}, i_{bn} and i_{bi} represent the op amp's noise sources. The combined output noise density is labeled "total."

FIGURE 36:   *Output Noise Densities.*

Comparing Figure 28 to Figure 36 shows that the white noise has been reduced. We also see the 1/f noise effect below 30 Hz.

Table 8 summarizes the information found in Figure 36 in convenient form. It is instructive to compare these results with those shown in Figure 25, Figure 28 and Table 5.

TABLE 8:   **NOISE VOLTAGE CONTRIBUTIONS TO THE OUTPUT**

Noise Source		E_{no} (μV_{P-P})
Thermal	R_1	0.78
	R_2	1.72
	R_3	2.89
	R_4	1.32
Op Amp	e_{ni}	5.31
	i_{bn}	0.36
	i_{bi}	0.45
Total		6.49

DESIGN OPTIMIZATION

With the basics of noise analysis and design under your belt, it is time to learn how to quickly and effectively optimize the noise performance of an op amp circuit.

Signal-to-Noise Ratio

The Signal-to-Noise Ratio (SNR) is one of the most common ways to decide if the noise in a circuit meets its design requirements. Usually, it is defined as the ratio of signal power (of a sine wave) to integrated noise power in decibels:

EQUATION 33: OUTPUT SNR

$$SNR = 20 \cdot log_{10}(V_{OUT}/E_{nout})$$

Where:

V_{OUT} = Sinusoidal output signal (V_{RMS})

E_{nout} = Integrated output noise voltage (V_{RMS})

SNR = Signal-to-Noise Ratio (dB)

In some applications, V_{OUT} is expressed in relation to its full scale range (V_{PK} or V_{P-P}). This will not be done in this application note.

Select a SNR value that supports the required accuracy for your design. Modify your circuit until it meets this SNR requirement. For a fixed output voltage, this is the same as minimizing the output noise.

Note:	Make your signal's full scale range as large as possible; this minimizes the cost and effort in reducing the output noise.

Reduce the Noise

When done properly, a design's noise performance will depend on only a couple of critical components. All of the other components can be chosen for other design goals.

PLACE THE GAIN UP FRONT

Place your high gain amplifier as close to the signal source (e.g., a sensor) as possible. Any noise sources after this gain will be divided by this gain; they should have little impact on the output noise. Noise sources before this gain, and in the high gain amplifier, will be critical to your design's success.

All of the amplifiers and other circuit elements after the high gain amplifier should be set to a gain as close to 1 V/V as possible.

FIND THE DOMINANT NOISE SOURCES

Any noise source that is at least half as large (in V_{RMS}) as the largest source should be considered to be a dominant source. This may appear to be a very loose requirement at first glance, but works very well in practice.

To illustrate this point, Table 9 illustrates how a larger noise source (E_{nout1}) and a smaller noise source (E_{nout2}) contribute to the total noise (Enout). The ratio E_{nout2}/E_{nout1} represents E_{nout2}'s magnitude relative to Enout1. The ratio E_{nout}/E_{nout1} represents how much larger E_{nout} is, compared to E_{nout1}, due to the contribution from E_{nout2}. When E_{nout2}/E_{nout1} is ½, or smaller, we can ignore E_{nout2}'s contribution within engineering accuracy (error less than 12%). Remember, the noise terms are the result of a Sum of Squares (followed by a square root operation).

TABLE 9: EFFECT OF SECOND NOISE SOURCE

E_{nout2}/E_{nout1}	E_{nout}/E_{nout1}
1/1	1.414
1/2	1.118
1/3	1.054
1/5	1.020
1/7	1.010
1/10	1.005

FILTER THE NOISE

Filter any noise with the lowest NPBW possible.

Place simple filters as close to dominant noise sources as possible; this helps when testing your design on the bench. A single real pole filter, using a resistor and a capacitor, is usually enough for most purposes.

Place more sophisticated filters further away from the source. This has the benefits of using one complex filter for many noise sources. This reduces the overall cost of active filter designs with low component sensitivities (changes in capacitors, resistors and op amp bandwidth have little impact).

Noise can alias into an ADC's baseband. Select an anti-aliasing filter with a BW much slower than the sample rate (e.g., ten times slower) to minimize this effect.

As we have seen before, a simple R-C lowpass filter at the very end, without a buffer, can minimize the op amps' contribution to the total noise. This filter can be placed at the input of an Analog-to-Digital Converter (ADC) as long as the last capacitor is much larger than the ADC's input sampling capacitor (to minimize gain error).

AN1228

Component Selection

There are a few simple rules that make it easy to select components that will meet your design goals.

RESISTORS

Resistors are usually chosen to be as small as possible at critical points in the design. The exception to this rule happens when the resistor acts like a current source in a circuit (e.g., the gain resistor in a transimpedance amplifier); the noise current is reduced by increasing the resistance (see Equation 10).

Avoid resistors that use carbon as the resistive material. They generate high levels of 1/f noise.

Use resistors with metal for the resistive material. Wire wound resistors typically have the best 1/f noise, but can cause high frequency circuit problems due to their parasitic inductance and capacitance. Metal film resistors have low 1/f noise and have good high frequency characteristics.

OP AMPS

Start your design with a general purpose part. Look at lower noise parts only after optimizing the rest of the circuit.

For high frequency applications (e.g., above 1 kHz), or applications that use auto-zeroed op amps, select the op amp based on its white noise (e_{ni} and i_{ni}).

For low frequency applications (e.g., below 1 kHz), also compare the 1/f noise performance (e_{ni} and i_{ni}). Compare the integrated noise between 0.1 Hz and 10 Hz (the Noise Voltage spec (μV_{P-P}) in Microchip's op amp data sheets). If that specification is not available in a data sheet, the noise spectrum plot will give the needed information. Compare the op amps' noise density at the same frequency in the 1/f noise region.

SUMMARY

This application note gives a simple overview of the noise theory used in circuit design. It is presented in a way to help like the reader's knowledge of statistics and circuit design to circuit noise design.

Many examples help build the reader's knowledge of the design process, how filters affect noise, how to combine multiple noise terms at the circuit's output, and optimizing a circuit's noise performance.

The topics cover what is needed in the majority of noise designs. Both white and 1/f noise are discussed. Manual analysis and computer simulations are used many times. Computer aided analysis is mentioned as a labor saving device.

After the body of this application note, there are selected references to the literature to help the reader find background material that covers this material well. Appendices with additional vocabulary and an overview of computer aids completes this application note.

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Related Application Notes

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Noise

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APPENDIX A: VOCABULARY

This appendix gives a brief list of common terms used in amplifier noise work. They are organized by topic so that their context is easier to grasp.

A.1 Spectral Densities

Power Spectral Density (PSD) is the frequency domain description of a noise source's statistical variation. Its units are (W/Hz) (sometimes converted to dBm/Hz). It is also related the noise's auto correlation function. It is also called Noise Power Density.

Noise Voltage Density (e_n) is the square root of PSD, normalized to a standard resistance (usually 1Ω). It has units of (V/√Hz). It is also called spot noise or noise per root Hertz.

Noise Current Density (i_n) is the square root of PSD, normalized to a standard resistance (usually 1Ω). It has units of (A/√Hz). It is also called spot noise or noise per root Hertz.

A.2 Spectral Shapes

White noise is a PSD that has a constant value over frequency. It is a mathematical convenience used to make system noise calculations simpler.

Broadband noise describes a noise source that is (nearly) white over a circuit's frequency range of interest. It isn't white, but appears to be white to that circuit.

Noise Power Bandwidth (NPBW) is mathematically convenient parameter used to describe how a circuit processes white noise. It has units of (Hz). It is the equivalent bandwidth of a brick wall filter that produces the same output noise as the actual circuit.

Excess Noise is any noise that exceeds the white noise level at low frequencies (only 1/f noise is discussed in this application note):

- 1/f noise, also known as flicker noise or pink noise
- $1/f^2$ noise, also known as red noise
- Random Telegraph Signal (RTS) noise, also known as burst noise or popcorn noise (has a spectral shape reminiscent of white noise filtered by a lowpass filter with a single real pole)

A.3 Integrated Noise

Noise Power (N) is the noise source's statistical variation. Its units are (W) (sometimes dBm).

Noise Voltage (E_n) is the square root of Noise Power normalized by standard resistance (usually 1Ω). It has units of (V_{RMS}, V_{PK} or V_{P-P}). When in units of V_{RMS}, it is also called the standard deviation.

Noise Current (I_n) is the square root of Noise Power normalized by standard resistance (usually 1Ω). It has units of (A_{RMS}, A_{PK} or A_{P-P}). When in units of A_{RMS}, it is also called the standard deviation.

A.4 Probability Density Functions

Many physical noise sources, but not all, have the Gaussian (or Normal) probability density function. They are said to be Gaussian Noise, or sometimes Additive White Gaussian Noise (AWGN). This noise is usually associated with random processes that fulfill the Identical and Independently Distributed (IID) assumption; it is the sum of a large number of statistically independent random variables with the same probability density function. The probability density function is:

EQUATION A-1:

$$p(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \cdot exp\left(\frac{-(x-\mu)^2}{2\sigma}\right)$$

Analog to Digital Converters (ADC) and Digital to Analog Converters (DAC) usually have their quantization errors modeled as random noise with a Uniform probability density function (the device noise at the inputs would be Gaussian, however). The probability density function is:

EQUATION A-2:

$$p(x; \mu, \sigma) = 1/(2\sqrt{3}\sigma), \quad |x-\mu| < \sqrt{3}\sigma$$
$$= 0, \quad \text{otherwise}$$

A.5 Figures of Merit

Signal-to-Noise Ratio (SNR) is the ratio of the signal power to the noise power. It is usually shown in units of (dB), although (V_{RMS}/V_{RMS}) or (A_{RMS}/A_{RMS}) are also acceptable. Sometimes the signal's full scale range is the numerator of the ratio, with units of (V_{PK} or V_{P-P}).

Other figures of merit not covered in this application note are:

- Noise Figure (NF) (dB)
- Noise Factor (F) (V/V)
- Noise Temperature (T_N) (K)

APPENDIX B: COMPUTER AIDS

While this application note emphasizes manual calculation and formulas, most design work uses computers.

B.1 Noise Simulations

Circuit noise simulations can be done as part of an AC simulation in SPICE simulators. The SPICE program was developed at UC Berkeley. Many SPICE derivative simulators are used in circuit design; the most popular for board level design is PSpice® (from Cadence®).

B.1.1 GENERAL REMARKS

The component models need to be correctly defined for noise simulations to give realistic results. Op amp macro models from Microchip are set up to work properly in PSpice. Resistors, diodes and transistors usually give the correct white noise (when the model is accurate). 1/f noise in diodes and transistors will not simulate correctly without special attention to the relevant parameters. The resistor model does not include 1/f noise; this can be added to the circuit using diodes and a dependent source, if needed.

You will need to define the input source and output circuit node before the noise analysis can run. SPICE produces an input referred noise vector (across frequency); it is referred to the chosen input source. SPICE also produces an output noise vector at the chosen node.

The noise results in different SPICE simulators come in two different forms: as noise voltage (current) density ($V_{RMS}/\sqrt{Hz}$) or as the square of the noise voltage (current) density (V_{RMS}^2/Hz). Check your simulator using resistor thermal noise; 1 kΩ of resistance, at +25°C, will give 4×10^{-9} $V_{RMS}/\sqrt{Hz}$ in the former case, and 1.6×10^{-17} V_{RMS}^2/Hz in the latter case.

Note:	PSpice produces noise in units of ($V_{RMS}/\sqrt{Hz}$). Other simulators may, or may not.

Use the simulator's plotting tool to determine which noise sources dominate and to improve the noise filtering and shaping.

B.1.2 CALCULATING INTEGRATED NOISE WITH PSPICE

To calculate integrated noise in PSpice, open its plotting utility (Probe) and add the following trace:

EXAMPLE B-1: PSPICE TRACE FUNCTION

```
sqrt(s(v(onoise)*v(onoise)))
```

This is the running integral (from 0 Hz to f, using the PSpice function `s()`) of the output noise (let's call it E_{n0f}) in units of ($V_{RMS}/\sqrt{Hz}$).

To obtain the integrated noise (E_{nout}) between f_L and f_H (see Figure 3), read E_{n0f}'s value at $f = f_L$ and $f = f_H$ (let's call these values E_L and E_H). The integrated noise is (for any spectral shape):

EQUATION B-1:

$$E_{nout} = \sqrt{E_H^2 - E_L^2}$$

To put this trace's data into a spreadsheet, click on its label found at the bottom left of Probe's screen. This selects this trace (the label changes color). Copy the data to Window's clipboard by typing the key sequence `Ctrl-C`. Paste the results into your spreadsheet. Following this sequence produced two columns of data; one frequency vector and one noise vector.

B.1.3 ESTIMATING NPBW WITH SPICE

Now that we can extract the integrated noise from our simulations, we can easily estimate a filter's NPBW. The following steps will make this more clear:

- Use a very large resistor (or 2 in parallel) as the noise voltage source
- Insert a buffer between the noise source and the filter's input
- Plot the output noise density (e_{nout})
- Calculate the integrated output noise (E_{nout}) from DC to infinity (a high enough frequency)
- Choose the e_{nout} value that represents the pass-band (at the chosen gain, H_M)
- Calculate the NPBW

EQUATION B-2: NPBW ESTIMATE

$$NPBW = (E_{nout}/e_{nout})^2$$

B.2 Using Symbolic Solver Engines

There are several places where a symbolic solver can speed up your noise analysis:

- Converting node equations to transfer functions
- Factoring transfer functions
- Expanding a magnitude squared transfer function into its Partial Fraction Expansion form
- Evaluating the definite integrals used for integrated noise (or NPBW)

Some popular tools are:

- Mathematica® (from Wolfram Research)
- Maple™ (from Waterloo Maple Software)
- Matlab® (from The MathWorks); use the Symbolic Math Toolbox™
- MathCad® (from Parametric Technology Corporation)

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